Split Decisions

Guidance for Measuring Locality Preservation in District Maps

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Guidance for Measuring Locality Preservation in District Maps

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Introduction

Redistricting can have an enormous impact on election outcomes.

Every ten years, states redraw their congressional and state legislative district boundaries to account for the shifting population, a process known as redistricting. Redistricting can have an enormous impact on election outcomes. By carefully drawing voters into specific districts, mapmakers can, for example, change the partisan makeup[1] of the U.S. House of Representatives by several seats or dramatically diminish minority representation.[2] New maps can have a similarly large impact on who is elected to state legislatures throughout the country.

In the aftermath of the 2020 election, voters split along party lines[3] when asked whether they trust the American election system. As recently as September 2021, 36% of Americans say that President Biden did not legitimately get enough votes to win the presidency.[4] Unfortunately, the usual process for how districts are drawn is unlikely to bolster voters' confidence that elections are free and fair. Typically, state legislatures enact maps as they do any other legislation. If one party controls the process, they can draw district lines to maximize their party's share of seats (i.e., partisan gerrymandering). Or both parties can collaborate to ensure safe districts for incumbents, who can cruise towards an easy re-election (i.e., bipartisan gerrymandering). All forms of gerrymandering undermine the idea that voters should choose their representatives, rather than the other way around – and therefore undermine trust in democracy.

As we enter the decennial redistricting period, there is bad news and there is good news.

First, the bad news. In 2019,[5] the U.S. Supreme Court held, in a 5–4 decision, that claims of partisan gerrymandering are not reviewable by federal courts. This is worrisome, because, as a result of the 2020 state legislative elections, the majority of

1 The Cook Political Report. (n.d.). *Road Map to Redistricting 2021-2022*. [perma.cc/8NML-AC3B]

2 Soffen, K. (2016, June 9). How racial gerrymandering deprives black people of political power. *Washington Post*. [perma.cc/BE36-KDLH]

3 Morning Consult. (n.d.). *How Voters' Trust in Elections Shifted in Response to Biden's Victory*. [perma.cc/FN22-FR2V]

4 Agiesta, J., Edwards-Levy, A. (2021, September 15). CNN Poll: Most Americans feel democracy is under attack in the US. *CNN*. [perma.cc/F658-7WRP]

5 Rucho v. Common Cause, 588 U.S. ___ (2019). [perma.cc/Q9N9-TCSM]

the population[6] currently lives in a state where one party will have full control of redistricting. In those states, one party will be free to maximize partisan gain, unencumbered by the other party or, now, by federal courts.

But the good news is that several large states,[7] including Colorado, Michigan, New York, Ohio, and others, have enacted redistricting reform since the last redistricting, ranging from bipartisan redistricting commissions, to citizens' commissions, to new statutory and constitutional fairness requirements. Additionally, new software has created avenues for the public to engage in the process, such as by submitting maps, evaluating maps,[8] and giving public input to redistricters, such as by indicating their communities of interest[9] (COIs) – geographically contiguous groups with shared cultural or economic characteristics that create common representational interests.[10]

When analyzing the effect that redistricting can have on representation, it is essential to determine which groups of voters are kept whole within a district, and which groups are split across districts. A large group of voters may have their electoral power needlessly diminished if they are concentrated within a single district – quarantining voter power that could otherwise be spread across multiple districts (i.e., "packing") – or if they are fractured across districts such that no representative prioritizes their interests (i.e., "cracking").

Groups of voters are not only defined by party, but also by race, ethnicity, language, economic interests, environmental interests, culture, history, shared government services, or other common legislative concerns.

This paper focuses specifically on measuring the extent to which a district map splits voters within a locality. In this paper, "localities" refers to contiguous geographic entities such as counties, cities, towns, and municipalities,[11] as well as COIs defined by the public.

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6 Wolf, S. (2021, August 11). *The Daily Kos Elections guide to how redistricting will unfold in all 50 states*. Daily Kos. [perma.cc/F2NK-S6JL]

7 Associated Press. (2020, March 5). More states to use redistricting reforms after 2020 census. *Associated Press*. [perma.cc/ZD8N-AD57]

8 Districtr. (n.d.). *Districtr*. [perma.cc/9U6T-Q2MU]
Dave's Redistricting. (n.d.). *About DRA*. [perma.cc/SS36-TUPB]
Princeton Gerrymandering Project. (n.d.). *Redistricting Report Card.* [perma.cc/JS7B-N8A5]
PlanScore. (n.d.). *PlanScore.* [perma.cc/7ZJH-MCX9]

9 Representable. (n.d.). *Representable*. [perma.cc/H2ZL-X652]

10 The Freedom to Vote Act, introduced by Senate Democrats in September 2021, defines communities of interest as "an area for which the record before the entity responsible for developing and adopting the redistricting plan demonstrates the existence of broadly shared interests and representational needs, including shared interests and representational needs rooted in common ethnic, racial, economic, Indian, social, cultural, geographic, or historic identities, or arising from similar socioeconomic conditions. The term communities of interest may, if the record warrants, include political subdivisions such as counties, municipalities, Indian lands, or school districts, but shall not include common relationships with political parties or political candidates." Freedom to Vote Act, S.2747, 117th Cong. (2021). [perma.cc/88C6-5QRU]

11 The U.S. Census Bureau refers to localities that provide governmental services as "incorporated places" and recognizes other unincorporated communities as "census designated places." Census Designated Places (CDPs) for the 2020 Census-Final Criteria, 83 Fed. Reg. 56290 (November 13, 2018). [perma.cc/AM6B-8G9H]

Keeping localities whole has several benefits to democracy. Accordingly, some states require that district maps preserve localities to the maximum extent possible. However, there is not a single best, commonly accepted way to measure the degree to which a district map splits localities. In this paper, we discuss the motivations for preserving localities and review current methods for measuring locality-splitting.

Some commonly-used metrics for measuring locality-splitting are entirely geography-based; they do not take into account where voters actually live. We recommend against using these metrics. We describe several population-based alternatives, introduce a new one which may have benefits, and provide additional guidance to those drawing or evaluating maps.

We hope that this guidance will enable redistricting officials and the public to select appropriate locality-splitting metrics, evaluate choices made in redistricting, and bring about fairer representation.

Groups of voters are not only defined by party, but also by race, ethnicity, language, economic interests, environmental interests, culture, history, shared government services, or other common legislative concerns.

Why keep localities whole?

Simplify election administration

Respecting local political boundaries makes it easier to administer elections. Election officials must create a unique ballot style for each set of different contests that a voter could be eligible for (including statewide, congressional, state legislative, municipal, and local races). If a district spans many different political boundaries, it increases the number of ballot styles, increasing the burden on election officials to ensure voters receive the right ballot. If election officials accidentally give some voters the wrong ballot, this can influence election outcomes and undermine confidence in the democratic process.

For example, in 2018, Virginia election officials accidentally assigned over a hundred voters to the wrong House of Delegates district.[12] This mistake may have changed the winner of the election and the party that controlled the chamber. By reducing the number of ballot styles required, the establishment of district boundaries that follow county and/or municipality lines can mitigate administrative problems by simplifying ballot assignment and tabulation.

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Inform voters

Keeping localities whole helps voters stay informed[13] about their representatives. A 2010 study[14] found that voters in better-preserved counties were more likely to be able to correctly name congressional candidates in their district. One possible explanation for this phenomenon is that TV advertising markets follow county boundaries, meaning that voters in preserved counties are more likely to only see news and advertisements about their specific congressional race. Another explanation is that voters get information from talking to their friends and neighbors, and that a county or a municipality may emerge as conversational shorthand for understanding local politics and representation.

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12 Branscome, J. (2018, February 8). Fredericksburg voters quietly drop lawsuit requesting new 28th District election. *The Free-Lance Star*. [perma.cc/7757-4TT9]

13 Stephanopoulos, N. (2012). Spatial Diversity. *Harvard Law Review*, 125, 1903-2010. [perma.cc/4SME-H6RX]

14 Winburn, J., & Wagner, M. W. (2010). Carving Voters Out: Redistricting's Influence on Political Information, Turnout, and Voting Behavior. *Political Research Quarterly*, 63(2), 373-386. [perma.cc/UQG9-EN7G]

Obstruct extreme partisan gerrymandering

When map drawers must preserve localities with pre-existing boundaries, it can make extreme partisan gerrymandering more difficult. The map that is best for a particular party has no inherent reason to preserve county boundaries, so map drawers who must avoid county splitting have less freedom to slice up voters in the optimal way for their desired partisan outcome. It is important to note that preserving counties does not, on its own, guarantee partisan fairness (Figure 1).

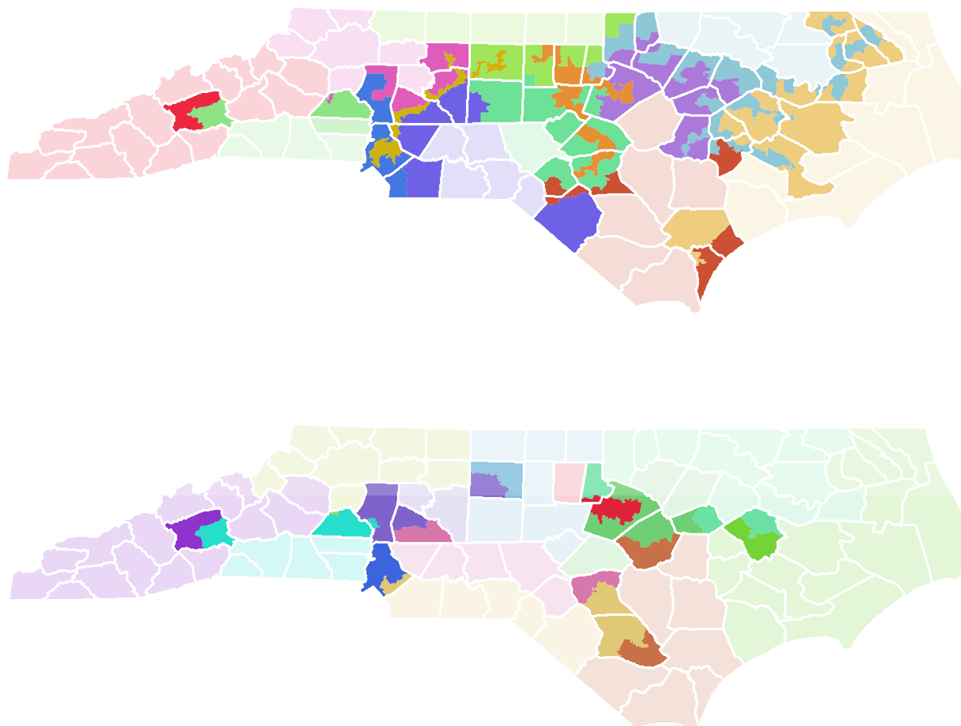

Figure 1. North Carolina counties (outlined) and congressional districts for the 113th Congress and the 115th Congress (colored; top and bottom, respectively). After Republicans' partisan gerrymander of North Carolina's congressional districts in 2011 (top), a federal court[15] tossed out the map, and the state legislature's 2016 revision (bottom) split about one-third as many counties. However, the new map performed as well or better at protecting Republican seats, even during the Democratic Party's strong 2018 midterms.[16] While limitations on county splitting makes extreme partisan gerrymanders more difficult, it is far from a sufficient constraint to prevent them. **Source:** U.S. Census Bureau.

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15 Harris v. Cooper. (2016). [perma.cc/QC5E-WMS7]

16 Adler, W. T., & Thompson, S. A. (2018, November 7) The 'Blue Wave' Wasn't Enough to Overcome Republican Gerrymanders. *New York Times*. [perma.cc/EC5P-3DAL]

Empower communities

Sometimes the reason to preserve a community is not because of shared political boundaries, but shared characteristics of the community's voters. Historically, map drawers have marginalized minority voters[17] (most often, Black voters in the Jim Crow South) by splitting them among many districts. In order to give minority voters more power to choose their representatives, the Voting Rights Act of 1965 requires that, in certain cases, map drawers must create minority "opportunity districts."[18]

Some states[19] give similar consideration to COIs. Examples may be a school district, a historically Cuban neighborhood, or a mining town. These communities stand to benefit from the power to choose their representative and lobby for specific legislation. In states that consider COIs, citizens have the opportunity to engage in the redistricting process by defining the communities that are important to them. Community-mapping platform Representable[20] allows communities to define their boundaries online and shares the data with redistricting officials. The mappability of COIs facilitates public testimony during the redistricting process and provides redistricting officials with the boundary data they need to preserve COIs.

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Statutory requirements

Given the important reasons to preserve localities, many states have statutory or constitutional requirements to avoid excessive locality splitting. A majority of states mandate that districts account for political boundaries (like counties and municipalities). A smaller but growing number of states require COIs to be kept whole when possible.[21] The Freedom to Vote Act, proposed by Democrats in the U.S. Senate in September 2021, would create federal requirements to preserve localities, including COIs, counties, and other political subdivisions.[22]

Sometimes, state redistricting law defines exactly what it means to respect locality boundaries. For example, Ohio's constitution provides detailed rules for how many counties and municipalities may be split and the manner in which they may be split.[23] But ambiguous provisions are much more common. An example is an Idaho law requiring, "[t]o the maximum extent possible, districts shall preserve traditional neighborhoods and local communities of interest."[24] This imprecise statement leaves a lot of room for judgment, especially due to inevitable trade-offs between locality preservation and other requirements like equal population and compactness of districts. It also raises questions

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17 Prokop, A. (2018, November 14). What is racial gerrymandering? *Vox*. [perma.cc/7CSN-V9R4]

18 Altman, M., & McDonald, M. (n.d.). *The Voting Rights Act*. Public Mapping Project. [perma.cc/RYF9-TUW6]

19 National Conference of State Legislatures. (2021, July 16). *Redistricting Criteria*. [perma.cc/S3KX-W87S]

20 Representable. (n.d.). *Representable*. [perma.cc/H2ZL-X652]

21 National Conference of State Legislatures. (2021, July 16). *Redistricting Criteria*. [perma.cc/S3KX-W87S]

22 Freedom to Vote Act, S.2747, 117th Cong. (2021). [perma.cc/88C6-5QRU]

23 Ohio Const. art XIX, pt. 2. [perma.cc/DE5L-AFR8]

24 Idaho Stat. § 72-1506. [perma.cc/9LSA-UF3B]

about how to define “traditional neighborhoods and local communities of interest.” Later, we will touch on this issue and the risks involved with requiring preservation of localities without pre-existing boundaries.

The metrics

Without specific guidance from redistricting law, it is tempting (and easy) to measure locality splitting by counting the number of localities that are split. However, this does not capture all the information about how many people are affected and how severely. As a result, several different splitting metrics appear in court documents and the redistricting literature.

Here we summarize five different metrics and introduce a sixth, describe the reasoning behind each one, and explain the similarities and differences in how they quantify locality splitting. In a later section, we will offer guidance on how a redistricting commission, journalist, or member of the public might choose a metric. For technical readers, we include mathematical definitions and detailed examples in the appendix. In addition, everything we discuss in this section is implemented in our GitHub repository.[25]

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Geography-based

Localities split

This is a very simple and commonly-used way to measure locality splitting: count the number of localities that are split into more than one district. In New Hampshire's two congressional districts (Figure 2) there are five split counties: Grafton, Belknap, Merrimack, Hillsborough, and Rockingham.

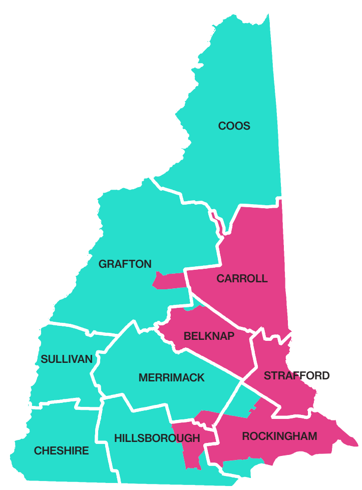

Figure 2. New Hampshire counties (outlined in white) and congressional districts for the 117th Congress (colored). Five counties are split into two districts. **Source:** U.S. Census Bureau.

25 Wachspress, J., Moffatt, C., & Adler, W. T. Metrics of locality splitting in political districting. (Version 0.2.1) [Computer software] [perma.cc/82DN-S35M]

Locality intersections

A shortcoming of the previous metric is that a locality that spans (i.e., intersects) three or more districts is treated exactly the same as a locality that spans just two districts. An alternative is to calculate the number of districts that intersect the locality. This way, splitting a locality into five districts is punished much more harshly than splitting it into two.[26] (See Figure 3.)

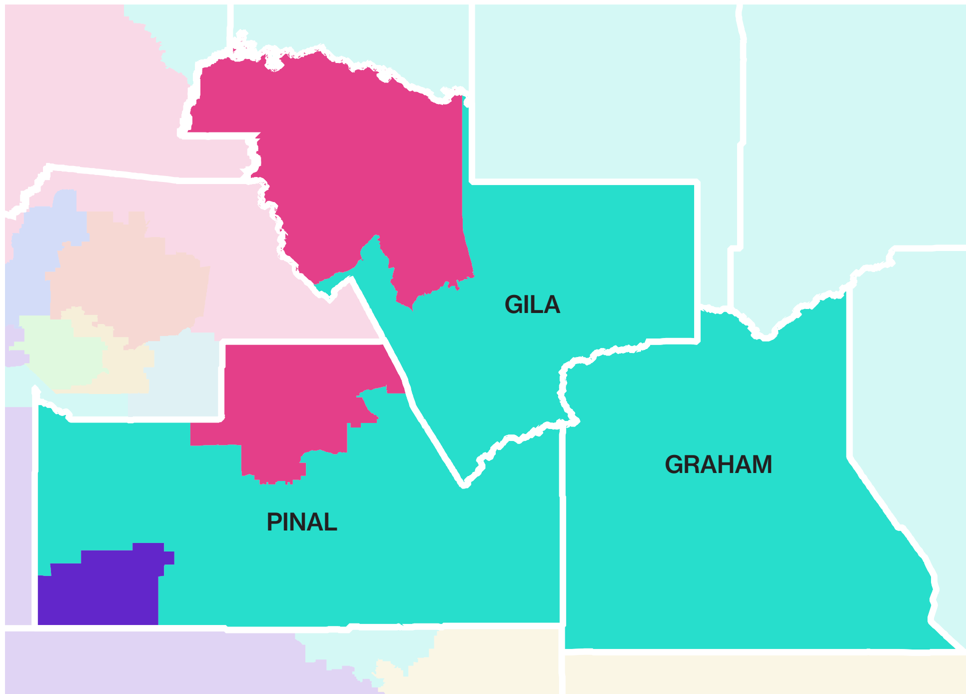

Figure 3. Partial map of Arizona counties (outlined) and congressional districts for the 117th Congress (colored). Graham County has one locality-district intersection, Gila County has two intersections, and Pinal County has three. **Source:** U.S. Census Bureau.

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Population-based

The previous two metrics treat every locality split the same, regardless of where people are actually located. This could under-penalize a split that separates a heavily-populated region (and therefore affects many people), and over-penalize a split involving a lightly-populated region (which affects fewer people). The previous metrics may also under-penalize splits that substantially divide a locality and over-penalize a split that peels off only a small fraction of people (Figure 4). For this reason, we suggest using metrics that work explicitly with population counts. These metrics measure the extent to which people in the same locality

26 Note that the number of contiguous geographical pieces does not matter for this calculation; for example, in the New Hampshire map above (Fig. 2), even though Rockingham County is split into three "pieces," the locality intersection metrics only counts the number of districts that the county intersects. Non-contiguous county pieces in the same district may be a sign of gerrymandering called "fracking" but is not one of the statewide splitting scores that we evaluate in this report. Cervas, J., Grofman, B., Horgan, T., & Freimer, R. (2021). [Fracking: A Contiguity-Related Redistricting Metric](). *SSRN*. [perma.cc/HU7C-2Q27]

are split by a districting plan and punish plans that affect more people.

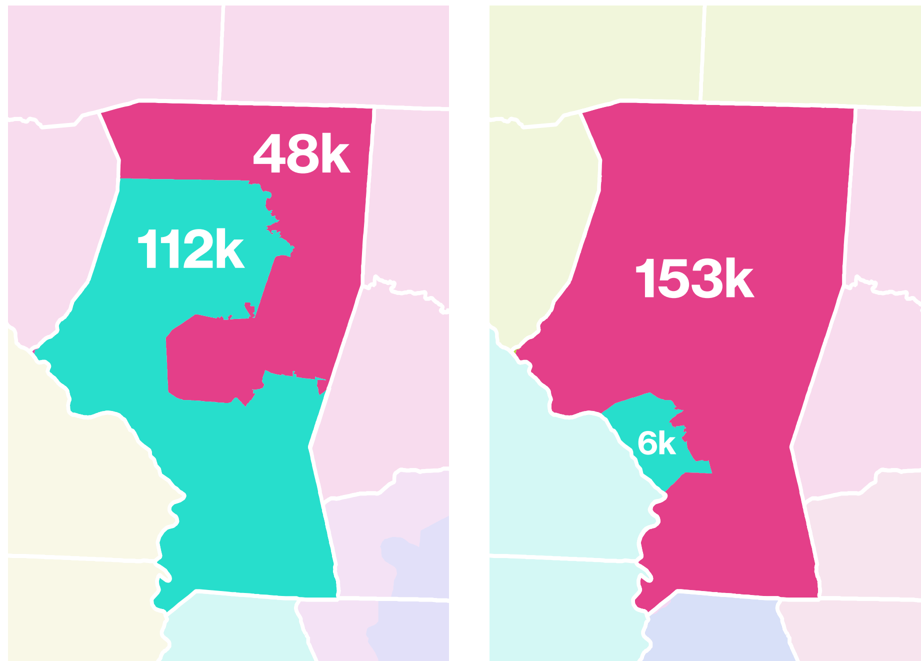

Figure 4. Partial map of North Carolina counties (outlined in white) and congressional districts for the 113th Congress and the 115th Congress (colored; left and right, respectively). Iredell County is split into two districts. Numbers indicate the population of Iredell County residing in each district. Geography-based splitting metrics score each of these county splits equally, despite the first map splitting Iredell County more significantly. **Source:** U.S. Census Bureau.

Effective splits

The "effective splits" metric was proposed for measuring COI splitting by Wang et. al. (2021)[27] and has roots in the political science literature of the 1970s.[28] It can be used to measure the splitting of any kind of locality, not just COIs. One way of thinking about this metric is: each person has a different perception of how split up their locality "feels," which depends on the proportion of the locality's people that are in that person's district. This metric attempts to aggregate each person's perception.

If a locality is split once, into two equally populated halves, each person feels as if the locality has been split once, for an effective splits score of 1. If it is split into three equal parts (each with 33.3% of the population), each person feels as if the locality is split twice, for an effective splits score of 2. If a locality is split into three parts constituting 80%, 10%, and

27 Wang, S., Chen, S. J., Ober, R., Grofman, B., Barnes, K., & Cervas, J. (2021). Turning Communities Of Interest Into A Rigorous Standard For Fair Districting. *SSRN*. [perma.cc/68FR-BMBE]

28 Laakso, M., & Taagepera, R. (1979). "Effective" Number of Parties: A Measure with Application to West Europe. *Comparative Political Studies*, 12(1), 3-27. [perma.cc/NHV3-F3GC]

10% of the population, the vast majority of people will feel relatively un-split and the effective splits score will be lower (in this case, about 0.5).

Conditional entropy

The "conditional entropy" metric proposed by Guth, Nieh, and Weighill (2020) quantifies the extra amount of information created by the district boundaries once the locality boundaries are known.[29]

The idea behind entropy in this case is to assign a "surprise score" to each person in the locality. If a person only knows the number of people from their locality in each district, how surprised will she be to learn which district she is in? If the locality is not split, no one will be surprised at all. If the locality is split into three parts constituting 90%, 5%, and 5% of the population, the people in the 90% part will not be surprised, but the people in the two 5% parts will be very surprised.

In order to quantify this surprise, conditional entropy divides 100% by the proportion of the locality's people in the same district. Since 100% divided by 90% is about 1.11 and 100% divided by 5% is 20, the people in the more populous part are much less surprised. For somewhat technical reasons related to quantifying information, the entropy metric then takes the base 2 logarithm of these numbers and reports the average across all people.

Square root entropy

Duchin (2018) proposed a slight modification of the conditional entropy metric in order to punish low-population splits more strongly.[30] The metric is known as "square root entropy" because the modification to the formula includes a square root sign.

Split pairs

With the goal of providing a simpler and more interpretable population-based metric than those in the literature, we are introducing the "split pairs" metric here. The metric calculates, among all pairs of people in the same locality, what proportion of them are split into two different districts. As a simple example, let's say that a small, rural locality called Alphabetville has 8 residents: A, B, C, D, E, F, G, and H. Suppose that A, B, C, and D are in one district, E and F are in another, and G and H are in yet another (Figure 5).

29 Guth, L., Nieh, A., & Weighill, T. (2020). Three Applications of Entropy to Gerrymandering. *arXiv*. [perma.cc/Y8WU-SMFM]

30 Duchin, M. (2018, February). *Outlier analysis for Pennsylvania congressional redistricting*. [perma.cc/A3AP-L84Z]

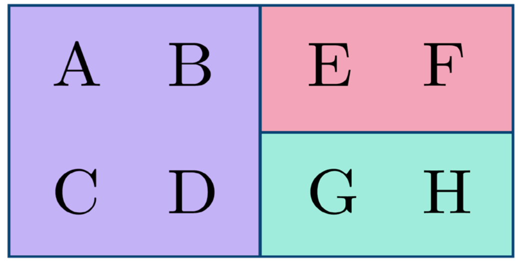

Figure 5. Map of a very small hypothetical locality, Alphabetville. Alphabetville's eight residents are divided into three separate districts (colored).

Then the following pairs of people are split into different districts:

AE, AF, AG, AH, BE, BF, BG, BH, CE, CF, CG, CH, DE, DF, DG, DH, EG, EH, FG, FH

while the following pairs are not:

AB, AC, AD, BC, BD, CD, EF, GH.

This makes 20 split pairs out of 28, for a split pairs score of 20/28=0.714. If all of the people were placed in the same district, there would be no split pairs, and the score would be 0.

The split pairs score can be summarized through the following hypothetical story: A random person does not remember his congressional district, so he picks a person randomly from his locality and asks what that person's district is. Then he guesses that he lives in the same district. What is the probability of guessing wrong? The split pairs metric.[31]

31 A slightly different version of this metric was formulated by Saxon (working paper). [perma.cc/9LSB-STRQ]

Why are locality-splitting metrics helpful?

Guide redistricting officials

Though state legislatures often prioritize partisan and incumbent interests in the redistricting process,[32] redistricting officials do sometimes adhere to best practices when determining which map to enact. In particular, this is typically the mandate of independent redistricting commissions. Redistricting officials should use a principled, population-based measure of locality splitting to assess possible maps. And to the extent that redistricting officials use sampling algorithms to provide or evaluate different options, the locality-splitting metrics can be used to influence which randomly generated maps the algorithm should accept or reject.[33]

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Legal tool for challenging bad maps

Locality-splitting metrics provide a legal tool for challenging redistricting plans, especially in states that require preserving localities "to the extent possible" but do not give more precise instructions. If a redistricting plan splits localities in an egregious way, a plaintiff should have no problem finding an alternative map that is better on *all* of the metrics in the literature. If this alternative also improves on other legally-prescribed attributes (e.g., compactness of districts), this adds to the body of evidence that the enacted map does not comply with redistricting law.

When evaluating how badly a map splits localities, it may be important to analyze why the localities are split. For example: Was a particular locality split to avoid a split somewhere else? Was the split necessary for balancing population across districts? Was it split to achieve a partisan goal, such as creating a competitive district – or maximizing gain for one party? Was it split to achieve a racial goal, such as creating a minority opportunity district – or cracking a racial group?

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Public engagement

An engaged public can put pressure on politicians to consider more than their own partisan and personal interests. This is especially relevant for COIs who want to advocate for their community to be kept whole. Through a collaboration with Representable,[34] members of the public will be able to see any districting plan's splitting scores for COIs. This will allow community members, coalitions of communities, and advocacy

32 Daley, D. (2016). *Ratf**ked: Why Your Vote Doesn't Count*. Liveright. [perma.cc/D2MB-NDRB]

33 DeFord, D., & Duchin, M. (2019). Redistricting Reform in Virginia: Districting Criteria in Context. *Virginia Policy Review*, 12(2), 120-146. [perma.cc/BVQ5-XYEE]

34 Representable. (n.d.). *Representable*. [perma.cc/H2ZL-X652]

organizations to experiment with alternative maps and hold redistricting officials accountable if their proposal splits communities excessively or unnecessarily.

Risks and limitations

Allowing the public to advocate for locality preservation or to draw their own COIs is not without its drawbacks. On the one hand, it allows regular citizens more input in the redistricting process than they have ever had. On the other hand, partisan actors can infiltrate this process, anonymously advocating for preserving particular localities only when it would favor their party. For example, in 2011, California Democratic House members reportedly coordinated efforts to influence the California Citizens Redistricting Commission in order to create a map that would protect Democratic incumbents.[35] This effort included advocacy for preserving specific counties and municipalities, and submissions from a fake nonprofit organization (with an accompanying Facebook page). In response, the Commission recommended that future Commissions "discuss and make decisions about the potential manipulation of the input process."[36] Reportedly, partisan actors have recently provided comments to the 2021 Commission without disclosing their affiliations.[37]

The threat of partisan actors "hacking" this process makes it even more important for regular citizens to be involved in a robust community-mapping process. This way, the public input phase actually represents real communities' interests. And it is also important that mapmakers or other entities do as much due diligence as possible in validating the identities and possible partisan affiliations of those submitting input. Representable vets its submissions by requiring mappers to provide detailed explanations of the characteristics (cultural, economic, historical, etc.) that unite their community.[38] They give this information to mapmakers and strongly encourage them to use it while assessing the validity of the submitted COIs. But redistricting commissions may want to go even further in ensuring that submitters' partisan affiliations, if any, are made clear.

35 Pierce, O., & Larson, J. (2011, December 21). How Democrats Fooled California's Redistricting Commission. *ProPublica*. [perma.cc/5TE2-76NG]

36 Aguirre, G. (2016, April). *Summary report and compilation of 2010 Commission actions and suggestions for future Citizens Redistricting Commissions*. California Citizens Redistricting Commission. [perma.cc/6ND4-EWAP]

37 Christopher, B., & Kamal, S. (2021, September 28). Between the lines: Hidden partisans try to influence California's independent redistricting. *CalMatters*. [perma.cc/8TY4-YYXP]

38 Representable. (n.d.). *Representable*. [perma.cc/H2ZL-X652]

Choosing (and using) a metric

This section provides some guidance on how to use locality-splitting metrics to assess statewide redistricting plans, including which metric to select and other choices to make. Many choices depend on the user's priorities, but we will make general recommendations where appropriate.

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Some metrics take population into account

As described above, the most commonly used methods for measuring locality splitting metrics – such as counting the number of localities split by a district plan – do not take population into account (Fig. 4). Population-based metrics instead measure the degree to which a district plan divides the population. Given that redistricting is fundamentally about the representation of people, we recommend the use of population-based metrics, except when redistricters are bound by statute to use other metrics. (Where statutes require the use of geography-based metrics, we recommend that state legislatures consider altering the law to allow redistricters to use population-based metrics.)

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Some metrics are easier to explain

Each metric quantifies something slightly different and may have benefits in different scenarios. But the importance of interpretability should not be understated. Advocates interacting with the press or lobbying elected officials require easily understandable metrics to get their point across. In certain circumstances, redistricting lawyers should be wary of bringing mathematical formulas into court, since judges may prefer standards that are simple and broadly applicable. (For example, Chief Justice John Roberts once referred to the "efficiency gap," a relatively simple mathematical measure of partisan fairness, as "sociological gobbledygook.")[39]

A redistricting commission may opt for a complicated metric that aligns with its priorities, but most others in the redistricting community will likely prefer using an easily explainable metric. To be sure, ease of explanation may bring tradeoffs: for example, while the geography-based metrics are the simplest to explain, their failure to consider people makes them much less desirable than the population-based metrics (unless state law dictates that this is how splitting should be measured). We think that the split pairs metric may strike an appropriate balance.

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39 Gill v. Whitford, 585 U.S. ___ (2018). [perma.cc/45MB-B2KP]

Population-based metrics agree most of the time

Among the population-based metrics, the particular choice matters but may not make a substantial difference in assessing maps. We analyzed the scores for Congressional and state legislative district maps before and after the 2010 redistricting (doing 123 comparisons), and found that any given pair of population-based metrics was in agreement 76–90% of the time on which map split counties worse.[40] (See appendix for more detail.)

The occasional disagreements between the metrics often occur when trying to discern the difference between similar-looking maps. In the cases where people are likely to notice or complain about locality splitting, the metrics will convey the desired information. For example, after court-ordered Congressional redistricting[41] in Florida (2015), Virginia (2016), North Carolina (2016) and Pennsylvania (2018), all of these metrics moved in the same direction on county splitting – they improved.

The reasons for the differences between the population-based metrics are subtle, and we will explain the two main factors here.

Some metrics punish more harshly splits that divide a larger fraction of people in a locality

A notable benefit of the population-based splitting metrics is that they treat locality splits differently depending on how many people they affect. Recall, the geography-based metrics do not differentiate a 96-4 split of a locality's people from a 70-30 split (Fig. 4). However, there is no "correct answer" for how much worse it is to split up more people. The splitting metrics all treat this question differently. If redistricting officials want to punish splits that affect a larger fraction of the locality's population *a lot more harshly*, they might consider using effective splits. If they would rather punish these splits just *a little more harshly*, square root entropy is the best choice. (See appendix for more detail.)

Punishment for dividing a large fraction of people	
Effective splits	*most harsh*
Split pairs	↑
Conditional entropy	
Square root entropy	↓
Localities split, locality intersections *(tie)*	*least harsh*

Table 1. Table indicating which metrics punish more harshly splits that divide a large fraction of people in the locality.

40 The two geography-based metrics agree with each other 87% of the time, and agree with the population-based metrics 60–79% of the time.

41 National Conference of State Legislatures. (2020, December 1). *Redistricting Case Summaries | 2010 – Present*. [perma.cc/7PF5-JXJA]

Deciding how to aggregate scores to the statewide level

Any method for assessing an entire districting plan will require some method for aggregating the metrics for each locality of a particular type (such as each county, municipality, or COI) into a single plan score. The choice of aggregation method reflects a judgment about how to penalize splitting in high-population localities versus low-population localities. For localities split, locality intersections, and effective splits, we recommend simply adding up all the splitting scores. For conditional entropy, square root entropy, and split pairs, we recommend taking a population-weighted average of the locality scores, so that each locality impacts the statewide score in proportion to its population.

We recommend this because the first three metrics measure "splitting events," treating each locality the same, while the logic behind the last three metrics operates on a "per person" level and only generalizes to the entire state if population weighting is used. (This is also consistent with the literature on conditional entropy and square root entropy.) The above analysis of pairwise metric agreement followed these conventions.[42]

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Recommendation

The geography-based metrics have a major shortcoming; they do not account for population. As such, we only recommend their use if state law dictates it (like in Ohio[43]). Among the population-based metrics, we will stop short of recommending a single one as the "best" way to calculate locality splits. In general, these metrics give similar results, so the choice is not particularly important, but they do reflect different choices about how much more to penalize splits that affect a large portion of a locality's population and splits that occur in more populous localities.

We proposed the new metric, split pairs, in an effort to provide a population-based metric that is as simple and interpretable as possible. It answers the relatively simple question: "How likely is a person to be in a different district than some randomly chosen person in her locality?" For the ease of explanation and the interpretable 0-to-1 scale, we believe this metric has promise. However, if a user wants to punish small splits, as in a map that takes tiny nibbles out of multiple localities, they may want to use a metric like square root entropy (Table 1).

42 It is worth noting that, among the types of localities that can be split by a district map, counties appear to be a special case: every person should reside in exactly one county. However, some people may not reside in a municipality, city, town, or other entity designated by the Census as an incorporated or unincorporated "place." And, of course, people may additionally reside in zero, one, or multiple COIs, depending on the set of COIs that the user chooses to include. This means that when aggregating scores across a state for any category of non-county locality, it is likely that some people will belong to more localities than others. Additionally, there may be a tension between preserving different kinds of localities that overlap. For instance, many municipalities span multiple counties, so it is impossible to preserve both municipalities and counties. U.S. Census Bureau. Census Designated Places (CDPs) for the 2020 Census-Final Criteria, 83 Fed. Reg. 56290 (November 13, 2018); List of U.S. municipalities in multiple counties. (2021, September 30). In *Wikipedia*.

43 Ohio Const. art XIX, pt. 2. [perma.cc/DE5L-AFR8]

Conclusion

With the August 12, 2021, release of the Census "redistricting file," redistricting season began.[44] As of publication time, only a few states have enacted or proposed any maps.[45] We expect the pace of map releases to increase as states reach their deadlines to implement new maps.[46]

Redistricters have always had many criteria to consider, with hard tradeoffs to be made.[47] This time around, fair redistricting may be even more complex than usual. With increased public access to software enabling meaningful engagement, and with requirements in 25 states to solicit public feedback, redistricters have a lot on their plates.[48] Clearly, legislators and commissions need quantitative tools to make sense of the decisions before them, including how to consider locality boundaries, whether pre-existing or submitted by the public. Additionally, the public benefits when tools to evaluate maps are publicly available. One such tool is the Princeton Gerrymandering Project's Redistricting Report Card, which incorporates several metrics, including the split pairs metric introduced here.[49]

For both the map-drawing process and the inevitable litigation over maps in the years to come, having principled metrics for evaluating map-making criteria is essential. A great deal of work has gone into creating methods for evaluating the partisan fairness of a map. Relatively less work has gone into creating or organizing methods for measuring locality splitting. We hope that this paper organizes and contextualizes that work, and provides a valuable resource for redistricters and courts alike.

44 U.S. Census Bureau. (2021, August 12). *2020 Census Redistricting Data Files Press Kit* [Press release]. [perma.cc/UMV2-LKSF]

45 FiveThirtyEight. (2021, September 28). What Redistricting Looks Like In Every State. *FiveThirtyEight*. [perma.cc/VA6E-XNM8]

46 National Conference of State Legislatures. (2021, March 29). *State Redistricting Deadlines*. [perma.cc/74HW-TGE3]

47 National Conference of State Legislatures. (2021, July 16). *Redistricting Criteria*. [perma.cc/S3KX-W87S]

48 Hernández, K. (2021, August 31). DIY Redistricting Allows Public to Draw Maps in More States. Stateline. [perma.cc/6SVH-4QC9]

49 Princeton Gerrymandering Project. (n.d.) Redistricting Report Card. [perma.cc/SR5B-BBF2]

Appendix

Formulas for population-based metrics

General definitions

For a given locality, let $D_1, D_2, \ldots, D_n$ be all the districts that have people in the locality.

For $i \in \{1, \ldots, n\}$, let V_i be the number of locality residents in D_i.

Define $V := \sum V_i$ (i.e., the total population of the locality) and $p_i := \frac{V_i}{V}$ (i.e., the proportion of locality residents in district i).

Recalling our Alphabetville example from the body of the paper (Fig. 6), we would have $V_1 = 4$, $V_2 = 2$, $V_3 = 2$, $V = 8$, $p_1 = \frac{1}{2}$, $p_2 = \frac{1}{4}$, and $p_3 = \frac{1}{4}$.

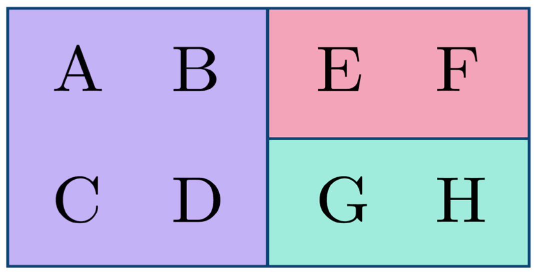

Figure 6. Map of a very small hypothetical locality, Alphabetville. Alphabetville's eight residents are divided into three separate districts (colored).

Effective splits

The formula for effective splits is given as:

$$\text{Effective splits} = \frac{1}{\sum_{i=1}^{n} p_i^2} - 1.$$

Note that if a locality is split into two equally-populated parts, this formula becomes $\frac{1}{\frac{1}{4}+\frac{1}{4}} - 1 = 1$ effective split. In general, a locality that is split into k equally-populated parts has k - 1 effective splits. Splits into unequal parts are punished more lightly.

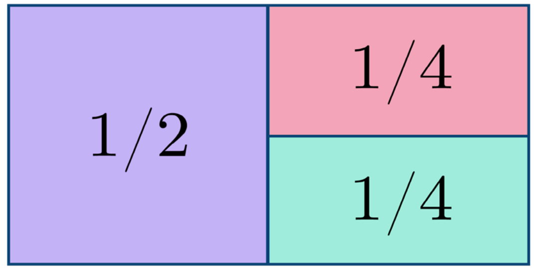

Figure 7. The proportion of the Alphabetville population in each district.

The people of Alphabetville are into districts that comprise $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{4}$ of the locality's population (Fig. 7). This gives an effective splits score of $\frac{1}{\frac{1}{4}+\frac{1}{16}+\frac{1}{16}} - 1 = \frac{10}{6} = 1.67$. To convert the effective splits scores from all of a state's localities into a single score for the entire districting plan, we recommend *adding the scores for each locality*. This is because this metric attempts to represent "splitting events," treating all localities equally.

Conditional entropy

The formula for conditional entropy is given as:

$$\text{Conditional entropy} = \sum_{i=1}^{n} p_i \log_2 \left(\frac{1}{p_i}\right)$$

Perhaps it is useful to think of this metric as an "average entropy per person." The amount of entropy that each person in district D_i contributes to the average is given by the formula $\log_2 \left(\frac{1}{p_i}\right)$ and shown in Fig. 7. Notice that if a person's locality is kept whole, then $\frac{1}{p_i} = 1$ for everyone, and the amount of entropy contributed to the average is $\log_2 1 = 0$.

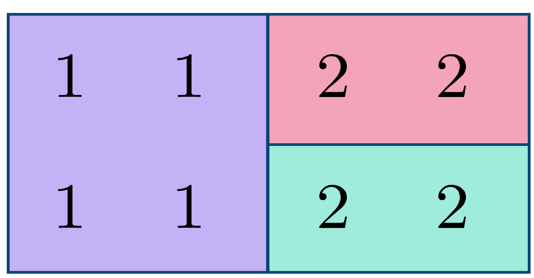

Figure 8. The entropy contributed to the average by each Alphabetville resident.

In Alphabetville, persons A, B, C, and D each have $p_i = \frac{1}{2}$, while persons E, F, G, and H each have $p_i = \frac{1}{4}$. Thus, the average conditional entropy per person is $\frac{1}{8}\left(4\log_2 2 + 4\log_2 4\right) = 1.5$.

To convert the conditional entropy scores from all of a state's localities into a single score for the entire districting plan, we recommend taking the *population-weighted average of the scores for each locality*.

Square root entropy

The formula for square root entropy is given as:

$$\text{Square root entropy} = \sum_{i=1}^{n} p_i \sqrt{\frac{1}{p_i}} = \sum_{i=1}^{n} \sqrt{p_i}$$

Thinking of this metric as an average per person, we note that each person in district D_i contributes $\sqrt{\frac{1}{p_i}}$ to the average. Note that this changes the score for non-split localities from 0 to 1.

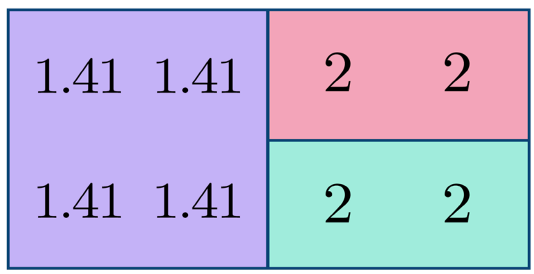

Figure 9. The square root entropy contributed to the average by each Alphabetville resident.

In the context of our example, persons A, B, C, and D each contribute $\sqrt{2}$ to the score, while persons E, F, G, and H each contribute $\sqrt{4} = 2$. This gives an average of 1.71 per person.

To convert the square root entropy scores from all of a state's localities into a single score for the entire districting plan, we recommend taking the *population-weighted average of the scores for each locality*.

Split pairs

The formula for split pairs is given as:

$$\text{Split pairs} = 1 - \frac{\sum_{i=1}^{n} \binom{V_i}{2}}{\binom{V}{2}}$$

where $\binom{K}{2}$ denotes $\frac{K(K-1)}{2}$.

Observe that the fraction is the number of pairs of people in the locality that are in the same district divided by the total number of pairs of people in the locality. By subtracting this from 1, we get the probability that a random person in the locality is in a different district from a random other person.

In Alphabetville, this expression becomes:

$$1 - \frac{\binom{4}{2}+\binom{2}{2}+\binom{4}{2}}{\binom{8}{2}} = 1 - \frac{6+1+1}{28} = \frac{20}{28} \approx 0.714.$$

To convert the split pairs scores from all of a state's localities into a single score for the entire districting plan, we recommend taking the *population-weighted average of the scores for each locality*.

***

More detail on the distinguishability of metrics

To determine how different these metrics are from each other, we checked how often they agreed about whether redistricting plans scored better or worse for county splitting after 2010 redistricting. To ensure an apples-to-apples comparison, we omitted the congressional maps where the number of districts changed after the 2010 Census. We also omitted comparisons between metrics when at least one of the metrics gave the same score to both plans. This includes, for example, congressional maps with only a single congressional district. The sample size was 123 for comparisons between population-based metrics and between 85 and 108 for comparisons involving the geography-based metrics (due to several occasions when the number of splits or intersections did not change).

	Locs. split	Loc. inters.	Eff. splits	Cond. entropy	Sqrt. ent.	Split pairs
Locs. split	1	0.87	0.78	0.62	0.60	0.68
Loc. intersects.		1	0.79	0.76	0.77	0.75
Eff. splits			1	0.85	0.76	0.85
Cond. entropy				1	0.89	0.90
Sqrt. ent.					1	0.79
Split pairs						1

Table 2. The frequency with which a pair of metrics agree on whether redistricting plans scored better or worse for county splitting after 2010 redistricting. Orange indicates geography-based metrics, and blue indicates population-based metrics.

Table 2 shows the frequency with which two metrics agreed on the direction of the change after redistricting. It indicates that, a substantial majority of the time, the population-based metrics agree with another on which of two maps is better, but that they occasionally disagree. There is much more disagreement between the population-based metrics and simply counting the number of splits. This occurs in part because counting splits is a relatively crude metric that ignores population and in part because of the different choices for how to aggregate locality scores into a statewide score. (This is why effective splits is the population-based metric that is most similar to the geography-based metrics.)

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More detail on punishing splits that divide a larger fraction of people in a locality

To see how each metric answers the question, “How much worse is it to separate more people in a locality?” we compared the penalty for a 90/10 split to the penalty for a 50/50 split (Table 3).[50] The ratios are in the rightmost column of the chart below. Higher ratios indicate the degree to which a metric punishes 50/50 splits more harshly than 90/10 splits. Notice the ratios of 1.0x for the geography-based metrics, which do not consider population.

	90/10 split penalty	50/50 split penalty	Ratio
Effective splits	0.22	1	4.6x
Split pairs	0.18	0.50	2.8x
Conditional entropy	0.47	1	2.3x
Square root entropy	0.26	0.41	1.6x
Localities split	1	1	1x
Locality intersections	1	1	1x

Table 3. The penalties each metric imposes on a 90/10 split or a 50/50 split, as well as the ratio of those penalties. Penalties are calculated by subtracting the score for an unsplit locality from a locality that is split 90/10 or 50/50. Orange indicates geography-based metrics, and blue indicates population-based metrics.

Accounting for populous localities and small districts

When a locality is much more populous than the required district population (as in Los Angeles County; see Fig. 8), there is no way to avoid splitting the locality.

50 Though there are several other splitting scenarios that may be worthy of future investigation as well. Duchin, M. (2018, February). *Outlier analysis for Pennsylvania congressional redistricting*. [perma.cc/A3AP-L84Z]

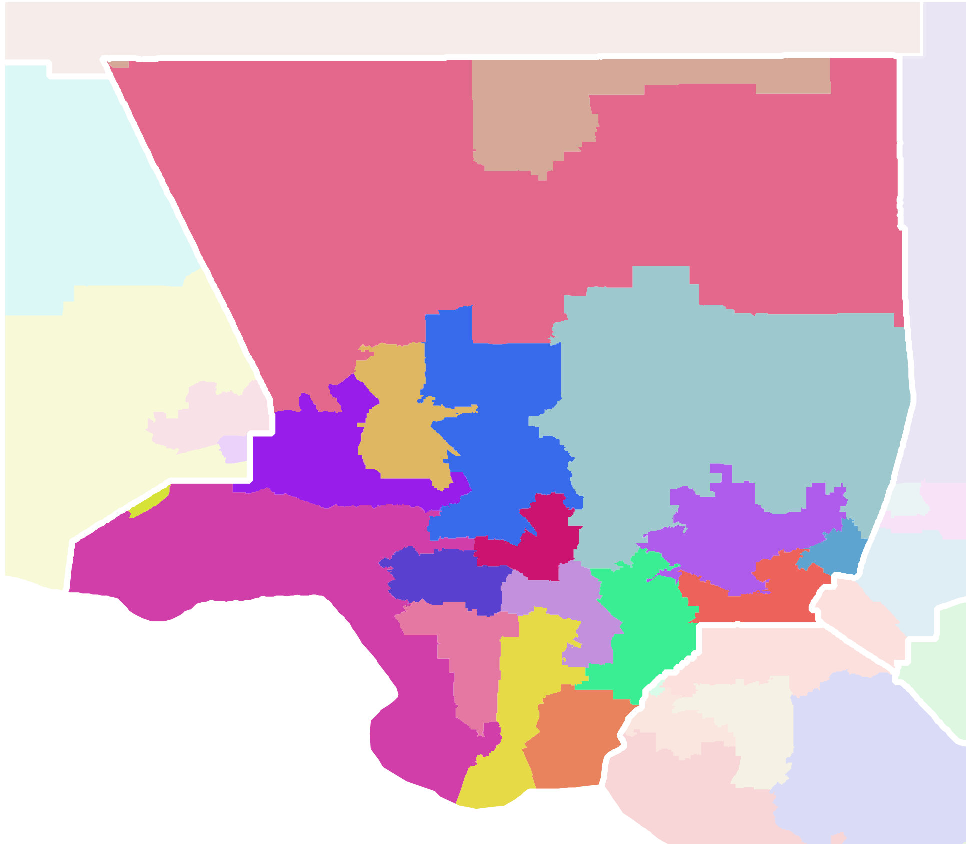

Figure 10. Los Angeles County, CA, (outlined in white) contained enough people for at least 14 congressional districts (colored) after the 2010 redistricting. The eventual district lines, shown here, were guaranteed to score poorly on the splitting metrics.

In these cases, some have proposed assessing maps on the extent to which districts are kept within a single locality. (In other words, swapping the roles of "locality" and "district" in all the splitting metrics.) In her report to the Pennsylvania governor, Duchin (2018)[51] calculated county-splitting scores both ways and took the average. This symmetric method has the advantage of explicitly considering the treatment of very populous localities. However, some of the reasons for preserving localities (e.g. election administration, voter engagement) don't apply to preserving districts. In practice, we found that it is uncommon for the two methods to disagree about whether a redistricting plan scores better than the state's previous map (see Table 4).

51 Duchin, M. (2018, February). *Outlier analysis for Pennsylvania congressional redistricting*. [perma.cc/A3AP-L84Z]

	Agreement
Locality intersections	100%
Effective splits	98%
Conditional entropy	97%
Square root entropy	93%
Split pairs	91%
Localities splits	91%

Table 4. The likelihood that each metric agrees with the symmetric version of itself on whether a map scored better or worse for county splitting after the 2010 redistricting. Orange indicates geography-based metrics, and blue indicates population-based metrics. Note that the "intersections" metric is already symmetric by definition, as it does not differentiate between localities and districts.

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